Lesson 6 : Prescribed Gaussian curvature problem, by A. Fardoun.

The Uniformization theorem asserts that every closed compact Riemannian surface admits a metric conformal to the initial metric with constant Gaussian curvature. In this course, we will study a generalization of the Uniformization theoerem that is the prescribed Gaussian curvature problem : let (M, g_0) be a closed compact Riemannian surface and let f be a given function on M (the prescribed function), does there exist a metric g conformal to the initial metric g_0 with Gaussian curvature equals to f ? We first introduce the prescribed Gaussian curvature equation and next we will give some analytic tools to solve this equation (such as Moser-Trudinger inequalities). Finally, we will study the prescribed Gaussian curvature problem depending on the sign of the Euler characteristic of the surface M and under conditions on the prescribed function. When M is the standard euclidean sphere, the problem is known as the Nirenberg problem and was solved by J.Moser in 1973 for even precribed functions.