Lesson 1 : Variational methods for elliptic problems, by E. Humbert

This course aims to give an overview of the basic tools to solve elliptic partial differential equations, and in particular the eigenvalue equation for the Laplace operator. We will start by working on an open set of $\mathbb{R}^n$ and then extend the results to Riemannian manifolds. The course will be divided into several parts :

1. **Sobolev spaces :** We will define Sobolev spaces. We will then give the main two theorems we will use later : the Sobolev embedding theorem and the Rellich-Kondrachov theorem.
2. **Elliptic operators :** We study second order elliptic operators with main goal to apply the results to the Laplace operator. We will present in particular the regularity theorems and the maximum principle.
3. **Variational method :** We present the variational method which allows us to solve some elliptic equations of the type $\Delta u(x) = f(x, u)$ with boundary conditions on open sets of $\mathbb{R}^n$.
4. **Variational method in Riemannian geometry :** In this last part, we will show how to adapt the previous results on Riemannian manifolds.

Except the last part which will require some basic knowledge in Riemannian geometry, the course will be self-contained.