Lesson 4. Quantitative Isoperimetric Inequalities, by A. Pratelli

The main aim of this course is to give a general overview on the old and recent results about quantitative versions of general inequalities of isoperimetric type. Such kind of estimates are basically “higher order” versions of classical inequalities. To explain this more precisely, we can just consider the example of the classical isoperimetric inequality : one knows that, among all the sets of given volume, those which minimize the perimeter are only the balls. The quantitative isoperimetric inequality tells something more : namely, that a set which is *almost* minimizing the perimeter must be *similar* to a ball, in a suitable precise sense. In this course we will present the basic history of this kind of problem and we will describe the main tools which are used to show such a result. We will initially focus on the “golden example” of the classical isoperimetric inequality, but we will conclude by giving other important examples.

Lesson 1 : Introduction and symmetrization tools

In the first lesson we will describe the history of the problem and the various steps forward which have been done through its resolution. Moreover, we will present the main technical tools which will be needed in the sequel, in particular the standard symmetrization results.

Lesson 2 : The N-symmetrization technique for the classical isoperimetric inequality

In the second lesson we will show how to use the tools presented before, in order to reduce the proof of the quantitative classical isoperimetric inequality to the case of the N-symmetric sets. The procedure will be presented quite in detail, to allow all the audience to understand the meaning of the different steps and the way to use of all the symmetrization tools.

Lesson 3 : The proof of the quantitative classical isoperimetric inequality

In the third lesson we will gather all the informations found in the first two, and we will show how to use them in order to conclude the proof of the main result. We will also discuss the overall strategy used, in order to point out the reason for the choice of this particular construction, and to understand which parts of the proof were really peculiar for the classical isoperimetric inequality, and which other parts were instead more general.

Lesson 4 : Other inequalities

This last lesson will be devoted to present several other inequalities of the same kind, namely, quantitative versions of other geometric and functional inequalities. We will not give the proof of any of them, but we will try to understand which properties can be deduced exactly as in the classical isoperimetric case, and which other properties need another different approach.