The Multi-players Nonzero-sum Dynkin Game in Continuous Time

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A Dynkin game is a game where the controllers make use of stopping times as control actions. Actually assume one has N players denoted by $\pi_1, ..., \pi_N$ and each of which is allowed, according to its advantages, to stop the evolution of a system. The system can be for example an option contract which binds several agents (players) in a financial market. So for $i = 1, ..., N$, assume that the player π_i makes the decision to stop the system at τ_i, then its corresponding yield is given by:

$$J_i(\tau_1, \cdots, \tau_N) := \mathbf{E}[X^i_{\tau_i} 1_{\{\tau_i < R_i\}} + Q^i_{\tau_i} 1_{\{\tau_i = R_i\}} + Y^i_{R_i} 1_{\{\tau_i > R_i\}}] \tag{0.1}$$

where $R_i := \min\{\tau_j, j \neq i\}$ and X^i, Q^i, Y^i are stochastic processes described precisely below. This yield depends actually on whether π_i is the first to stop the evolution of the system or not. So the main problem we are interested in is to find a Nash equilibrium point (hereafter NEP for short) for the game, i.e., an N-uplet of stopping times $(\tau_1^*, ..., \tau_N^*)$ which satisfy, for any $i = 1, ..., N$ and any $(\tau_i)_{i=1,N}$,

$$J_i(\tau_1^*, \cdots, \tau_N^*) \geq J_i(\tau_1^*, ..., \tau_{i-1}^*, \tau_i, \tau_{i+1}^*, ..., \tau_N^*).$$

Roughly speaking, a NEP is a collective strategy of stopping for the players which has the feature that if one of them decides unilaterally to change its strategy of stopping then she/he is penalized.

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In the case when $N = 2$ and $J_1 + J_2 = 0$, the game is called of *zero-sum* type and the corresponding NEP is called a *saddle-point* for the game. Otherwise the game is called of *nonzero-sum* type.

Nonzero-sum Dynkin games in continuous time were introduced by Bensoussan and Friedman [1] in the mid-seventies, in the Markovian framework and in connection with partial differential equations. Since, they have attracted few research activities in comparison with the zerosum setting. Besides, in all those works, is considered only the case of two players (even in [1]) under rather tough assumptions. Actually except in a recent paper by S.Hamadène and J.Zhang [2], authors assume that the processes Y^i of (0.1) are supermartingales. Mainly for reasons related to the methods which are employed. In [2], this latter condition is removed but the authors deal only with the case of two players.

Then the main objective of this paper is to study the continuous time nonzero-sum Dynkin game in the case when there are more than two players and general stochastic processes X^i, Y^i and Q^i, $i = 1, ..., N$. So far this problem is left open and none of those two aspects of this game problem is considered yet. As in [2], we also study the setting where the processes Y^i are no longer supermartingales. We show that the game has a Nash equilibrium point under rather minimal assumptions. ■

References

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