# Introduction to Delaunay dual quantization with applications to discretization schemes of BSDE's

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In this mini-course we will develop several aspects of optimal quantization in connection with the discretization of BSDE'ss. We will focus on *dual quantization*, a new notion of quantization recently introduced and studied in a series of papers [6, 8, 7] which produces smoother approximations than Voronoi quantization since it relies on an interpolation on the Delaunay triangulation rather than a projection following the nearest neighbour rule.

We also establish new *a priori* error bounds which improves those obtained for usual quantization based discretization schemes, *e.g.* in [2, 1] or [?]. We will first apply these results to Bermuda options and as a second step to BSDE's, possibly with reflection

- Regular Voronoi quantization of a random vector. Function approximation by stepwise constant function on the Voronoi tessellation of a grid.
- Dual Delaunay quantization of a random vector. Function approximation by continuous stepwise affine function on the Delaunay triangulation of a grid (see [6]).
- First example of numerical application : pricing multi-asset Bermuda options on a quantization tree. Delaunay *vs* Voronoi? Improved error bounds (see [7]).
- Optimal Voronoi and Delaunay quantization. Optimal quantizers, Quantization rate (Zador's Theorem), the curse of dimensionality (see [4] and [8]) and how to beat it (spatial Romberg extrapolation).
- How to get optimal quantization by simulation? (see [3])
- Application to the discretization of BSDE and reflected BSDE's .
- Toward hybrid methods: regression method on Voronoi tessellation/ Delauany triangulation (see [5]).

## References

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